and by equation (2), therefore,

$$\frac{dy}{d\theta} = - R \sin \theta \ .$$

In the same fashion,

$$\frac{dx}{d\theta} = - R \cos \theta \ ,$$

and the final expressions for x and y are obtained by integration:

$$\left.\begin{aligned} x(\theta_s) &= \int_{\theta_s}^{\theta_o} R(\theta) \cos \theta \, d\theta \ , \\ y(\theta_s) &= \int_{\theta_s}^{\theta_o} R(\theta) \sin \theta \, d\theta \ . \end{aligned}\right\} \qquad (3)$$

The function $y(x)$ is thus available in parametric form, the parameter being the cutting angle θ_s at the endpoint of the integration.

The impingement angle θ_o can lie between 0° and 180°. The local angle θ_s falls to 0° at the deepest point of the cut. θ_s cannot fall below 0°, because negative θ_s would mean the rock somehow were reconsolidating and filling up the cut. It follows that the depth h of the cut is given by the formula

$$h = \int_0^{\theta_o} R(\theta) \sin \theta \, d\theta \ , \qquad (4)$$

which is the main result of this section. The task remains for dynamics to determine the local radius of curvature R as a function of angle θ.

One further geometrical assumption will be made to simplify the fluid dynamics, namely that the depth d of the jet stream is everywhere small compared with the radius of curvature R:

$$d \ll R \quad \text{or} \quad d_o \ll h \ . \qquad (5)$$

The two inequalities are essentially equivalent. The theory is tailored to deep cuts, but the predictions agree with data measured by Olsen and Thomas down to $h/d_o \approx 1$. Shallower cuts give way to pitting and spalling, so the theory seems valid over the whole regime where the notion of "cutting" itself is warranted.